Math Links

PROFICIENCY CHALLENGES
GRADE 7
SETS 9-12

Proficiency Challenges are sets of interesting, mixed-topic problems. It may take a lot of time to complete each set, so consider doing only one or two parts at a time.

PROFICIENCY CHALLENGE 9

1. Walter spent time analyzing the equation $x + 7 = x + 8$.
   Finally he said, "I don't think this equation has a solution."
   Explain why Walter's conclusion is correct.

2. Wendy tried different methods to solve the equation $2(3 + x) = 2x + 6$.
   After a while, she became exasperated! "It doesn't seem to matter what x is! I can put any number in there and the equation will be true."

   Explain why Wendy is correct.

3. In 8th grade, you'll learn more about equations like those above. In problem 1, the equation is said to have "no solutions" because there is no value for x that will make the equation true.

   Create another equation that doesn't have a solution and justify your answer.

   In problem 2, the equation is said to have "infinitely many solutions" because any value of x will make the equation true.

   Create another equation that has infinitely many solutions and justify your answer.

4. Consider three consecutive numbers. A total is found by adding the first number, plus three times the second number, plus six times the third number.

   Which of these expressions could represent the total? Choose all that apply.

   A. $x + 3x + 6x$
   B. $x + 3(x + 1) + 6(x + 2)$
   C. $x + (3x +1) + (6x + 2)$
   D. $x + 3x + 3 + 6x + 12$

   Explain your reasoning.

   Find three consecutive numbers that meet the condition above if the total is 95.

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PROFICIENCY CHALLENGE 9 (Continued)

5. Read each of the following situations. For each situation, determine if one, both, or neither of the equations could be used to answer the question.

$$30 + x = 240 \qquad\qquad 30x = 240$$

- A theater can seat 240 customers. The theater contains 30 rows with the same number of seats in each row. How many seats are in each row?

- There are 30 red fish in the aquarium. The total number of fish in the aquarium is 240. How many non-red fish are there in the aquarium?

- Philip had 30 baseball cards in his collection. He gives some to his younger sister for her birthday. Her sister now has 240 cards in her collection. How many cards did Philip give here?

- There are 30 bottles of soda in a case. The manager of the restaurant needs to order 240 sodas total. How many cases should he order?

- Raul and Sal volunteered at a community beach clean-up day, Raul collected 30 pieces of trash. Together they collect 240. How many pieces of trash did Sal collect?

6. In the expressions below, m and n are both integers. Find values of m and n for which the expression in column A is equivalent to the expression in column B. Answers for one part have no relationship to other parts.

	A	B
a.	$m(2x + 3)$	$-6x + n$
b.	$m(2x + 3)$	$6x + n$
c.	$\dfrac{4x-8}{m}$	$-nx - 1$
d.	$\dfrac{4x-8}{m}$	$nx - 1$

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PROFICIENCY CHALLENGE 10

1. Daniel needs to pick up some fruit for his morning smoothie. He only has $5, and he needs to buy one pineapple and as many bananas as possible.

    - Each pineapple costs $2.79.
    - Each banana costs $0.38.

    a. Assuming there is no tax, write an inequality that represents the number of bananas, b, that Daniel can buy.

    b. How many bananas can Daniel buy?

    c. Make a graph that represents the number of bananas Daniel can buy on a number line.

    d. Rachel says that Daniel has enough money to buy 5.81 bananas. Explain why Rachel's math is correct but her answer might not make sense in this case.

2. Marshal is making trail mix with a two boxes of raisins, a small bag of chocolate chips, and as many peanuts as he can afford. In total, the trail mix needs to cost less than $10.

    - A box of raisins costs $2.37.
    - A small bag of chocolate chips costs $1.86.
    - Peanuts cost $2.64 per pound.

    a. Assuming there is no tax, write an inequality that represents the number of pounds of peanuts, p, that Marshal can buy.

    b. Make a graph that represents your solution. How does this graph compare with your graph from problem 1?

    c. Rachel (from the problem above) says that Marshal can only buy 2 pounds peanuts because he does not have enough money to buy another pound. Critique Rachel's reasoning.

PROFICIENCY CHALLENGE 10 (Continued)

3. "I'm confused," said Rachel. "In problem 1, the answer had to be a whole number because Daniel couldn't buy part of a banana. But in problem 2, you're telling me that Marshal can buy parts of peanuts!"

   a. Explain to Rachel why some contexts permit only whole number solutions (like the context in problem 1) and why some contexts permit non-whole number, rational solutions (like the context in problem 2).

   b. Express one more context when solutions must be whole numbers and one more context when solutions can be non-whole number, rational numbers.

4. Write expressions for each of the following.

   - The area of a rectangle that has a length of $2x + 4$ and a width of 2.
   - The area of a rectangle that has a length of $2x + 2$ and a width of 2.
   - The perimeter of a rectangle that has a length of $2x$ and a width of 2.
   - The perimeter of a square with a side length $x + 1$.

   Which of the expressions above are equivalent for all values of x?

5. Carlos says "I think of a number, add 5 and then multiply by 8. My answer is 96. What's my number?"

   a. Which of the following equations below will provide the answer to Carlos' question? Choose all that apply, and then explain your reasoning.

      - $8x + 5 = 96$
      - $8(x + 5) = 96$
      - $x + 40 = 96$
      - $8x + 40 = 96$

   b. What is Carlos' number?

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PROFICIENCY CHALLENGE 11

1. A t-shirt company makes custom t-shirts in four sizes: S, M, L, and XL.

   The following table shows a breakdown of the costs to make one t-shirt for each size.

   a. Calculate the total cost for each shirt size.

Size	Amount of Fabric (in yards)	Cost of Fabric (per yard)	Cost of Labor	Total Cost for each shirt
S	2.5	$2	$3	
M	3	$2	$3	
L	3.5	$2	$3	
XL	4	$2	$3	

   b. Make a graph of the data showing the amount of fabric on the *x*-axis and the total cost for each shirt on the *y*-axis.

   c. Does the relationship appear to be proportional? Explain.

   d. If you were to draw a "trend line" through the points, where does that line cross the *y*-axis? What might this value represent in the context of the problem?

   e. What is the increase in cost per increase in 1 yard of fabric? How is this value represented in your graph?

2. A newspaper recently published a story that was titled: "Should People with Bigger Feet Pay More for Shoes?"

   a. Based on what you've learned so far about ratios and proportional relationships, and what you examined in problem 1, what evidence or arguments might this article be trying to make about why people with bigger feet should pay more for shoes?

   b. What might be a counterargument for why people with bigger feet should NOT pay more for shoes?

PROFICIENCY CHALLENGE 12

1. Bruno won $5,000 in the lottery. He wants to give 50% of this money to family members. So he decides to give 10% of the $5,000 to his mom. He gives 10% of the remaining money to his dad. He then gives 10% of the remaining money to his sister. He then gives 10% of the remaining money to his brother. He then gives 10% of the remaining money to his nephew.

   Bruno says "I have given 10% of my money to five different family members, therefore I have given 50% of my money away."

   Explain why Bruno's conclusion is incorrect using numbers and words.

2. In 2010, Manuela invested $400 at an annual interest rate of 5%. In 2013, she invested $700 at an annual interest rate of 6%.

   How much will her total investments be worth in 2023?

3. A department store advertises a sale that allows customers to choose to receive a $12 discount on any item or a 12% discount on any item.

   Brian wants to buy a $70 vacuum cleaner. Nick wants to buy a guitar for $160. Each of them asks you which option they should each choose in order to save the most money.

   a. Explain to each of them what their choice should be.

   b. After a few more friends ask you for your advice, you start to get annoyed at having to do the calculations every time. What could be a general rule of advice that you could tell your friends that would allow them to decide which option to choose without having to do any calculations?

4. Malik is offered a job selling cars. The company offers him a choice. He can receive an annual salary of $50,000 or he can make a commission of 5% on his sales.

   a. If Malik wants to make the most money he can, what variables should Malik consider before making his decision?

   b. At the end of the year, Malik says "Both options would have given me about the same amount of money." What was the total value of his sales? Explain your reasoning.